Lecture 6 Summary

Wed 06/08/14

Vocabulary

* base
* connected
* disconnected
* continuous

Examples

PREIMAGES

Let $f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^2 + y^2$

(1) If $A = [0,1]$, $f^{-1}(A) = \{(x,y) \in \mathbb{R}^2, x^2+y^2 \leq 1\}$ = closed unit disc

(2) If $B = (1,2)$, $f^{-1}(B) = \{(x,y) \in \mathbb{R}^2, 1 < x^2+y^2 < 2\}$ = open annulus

Homework

Exercise 1:

- Let $\mathcal{B}$ be a collection of subsets from set X, such that
  (1) X is a union of sets from $\mathcal{B}$
  (2) if $B_1, B_2 \in \mathcal{B}$ then $B_1 \cap B_2$ is a union of sets from $\mathcal{B}$

  Then $\tau = \{$all unions of sets from $\mathcal{B}\}$ is a topology on X with $\mathcal{B}$ as a base.

Exercise 2:

- Let (X,d) be a metric space. Show that
  $\mathcal{B} = \{$all open balls in $X\}$
  satisfies the conditions (1), (2) in Exercise 1, so $\mathcal{B}$ is a base for the metric space topology.

Exercise 3:

- Let $(X, \tau_X), (Y, \tau_Y)$ be topological spaces. Show that
  $\mathcal{B} = \{U \times V : U \in \tau_X, V \in \tau_Y\}$
  is a base for the product topology on $X \times Y$ using Exercise 1.

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